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Reg. No.....

Name.....

# B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2018

## Fourth Semester

Complementary Course—STATISTICAL INFERENCE

(Common for B.Sc. Mathematics, Physics and B.Sc. Computer Applications)

[2013 Admission onwards]

Time : Three Hours

Maximum Marks : 80

## Part A (Short Answer Questions)

*Answer* **all** *questions.*
*Each question carries* 1 *mark.*

1. Define unbiasedness.
2. Define point estimation.
3. What do you mean by confidence internal ?
4. Define type 1 error.
5. Define level of significance.
6. How is large sample test different from small sample test ?
7. Give the test statistics for the equality of two population variances.
8. Define sufficiency.
9. Give an example of an estimator which is both unbiased and consistent for population mean.
10. State Cramer Rao inequality.

(10 × 1 = 10)

## Part B

*Answer any* **eight** *questions.*
*Each question carries* 2 *marks.*

11. What is meant by one-way ANOVA ?
12. Define sufficiency. Give an example of a sufficient estimator.
13. Define : (i) Simple ; (ii) Composite hypotheses. Give examples for each.
14. Define : (i) Statistic ; (ii) Sampling distribution.

**Turn over**

15. Define '$t$' statistic for testing $H_0 : \mu = \mu_0$ based on a random sample from $N(\mu, \sigma^2)$.

16. Write down the statistic for Chi-square test of goodness of fit.

17. Give the 95% confidence interval for the population proportion based on a large sample proportion P.

18. If the mean of a random sample of size 100 from $N(\mu, \sigma^2)$ is 30 and $\sigma^2 = 25$, find 95% confidence interval for $\mu$.

19. If $X_1, X_2, \ldots\ldots X_n$ is a random sample from a normal population $N(\mu, 1)$, show that $\frac{\Sigma X_i^2}{n}$ is an unbiased estimate of $\mu^2 + 1$.

20. For the p.d.f. $f(x, \theta) = \theta x^{\theta-1}, 0 < x < 1, \theta > 0$, find Cramer-Rao lower bound for the variance of unbiased estimate of $\theta$.

21. Define consistency and give a set of sufficient conditions for consistency.

22. What is meant by a moment estimator ?

$(8 \times 2 = 16)$

## Part C

*Answer any* **six** *questions.*
*Each question carries* 4 *marks.*

23. Show that while sampling from $N(\mu, \sigma^2)$, $\chi^2 = \frac{ns^2}{\sigma^2}$, where $s^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$ is not unbiased, but consistent for $\sigma^2$.

24. Explain the method of maximum likelihood estimation.

25. Let $(20, 22, 21, 24, 21)$ be a random sample for $N(\mu, \sigma^2)$. Obtain 95% confidence interval for $\sigma^2$.

26. Let $(x_1, x_2, \ldots\ldots s_9)$ be a random sample from $N(\theta, 25)$. If for testing $H_0 : \theta = 20$ against $H_1 : \theta = 26$, the critical region is given by $W = \{x \mid \bar{X} > 23.266\}$, find the size of the C-R and power.

27. Obtain the MLE of $\theta$ in the p.d.f. $f(x,\theta)=(1+\theta)\,x^{\theta}, 0<x<1$, based on a sample of size $n$. Examine whether the estimate is sufficient of $\theta$.

28. Obtain the MLEs of $(a,b)$ if $X \sim U(a,b)$ with $f(x)=\frac{1}{b-a}, a<x<b$.

Find the MLEs of $(a,b)$ if the observations are 2, 4, 8, 6, 7, 10, 11, 10, 7, 6.

29. Explain Chi-square test of independence of two attributes.

30. Explain the test procedure for the equality of two population proportions if large samples are being taken from them.

31. Obtain a sufficient estimator for $\mu$ and $\sigma^2$ in $X \sim N(\mu, \sigma^2)$.

$(6 \times 4 = 24)$

## Part D

*Answer any* **two** *questions.*
*Each question carries* 15 *marks.*

32. If $X \geq 1$ is the C-R for testing $H_0 : \theta = 2$ against the alternative $H_1 : \theta = 1$, on the basis of a single observation from the population $f(x,\theta)=\theta e^{-\theta x}, 0<x<\infty$, obtain the probability of type I error and power.

33. Explain (i) *t*-test for equality of means and (ii) F-test for equality of variances of two normal populations.

34. Fit a binomial distribution to the following data and test for its goodness for fit, under the assumption that male and female births are equally probable :

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Boys | : | 0 | 1 | 2 | 3 | 4 | 5 |
| Girls | : | 5 | 4 | 3 | 2 | 1 | 0 |
| No. of families | : | 12 | 20 | 36 | 18 | 10 | 4 |

35. (a) It is the claim more IAS selections are made from cities rather than rural areas. On the basis of the following data, do you uphold the claim $\alpha = 0.05$.

| IAS | Selected | Not Deleted | Total |
|---|---|---|---|
| City | 500 | 200 | 700 |
| Rural | 100 | 30 | 130 |
| Total | 600 | 230 | 830 |

(b) A standard examination has been given for several years with $\mu = 70$ and $\sigma^2 = 90$. A school using this examinations for the first time conducted it to a group of 25 students who obtained mean marks 71 and sample variance 12. Is there reason to doubt the process variance increased ($\alpha = 0.05$).

$(2 \times 15 = 30)$